Satellite Orbits and Orbital Mechanics

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Space Systems Laboratory

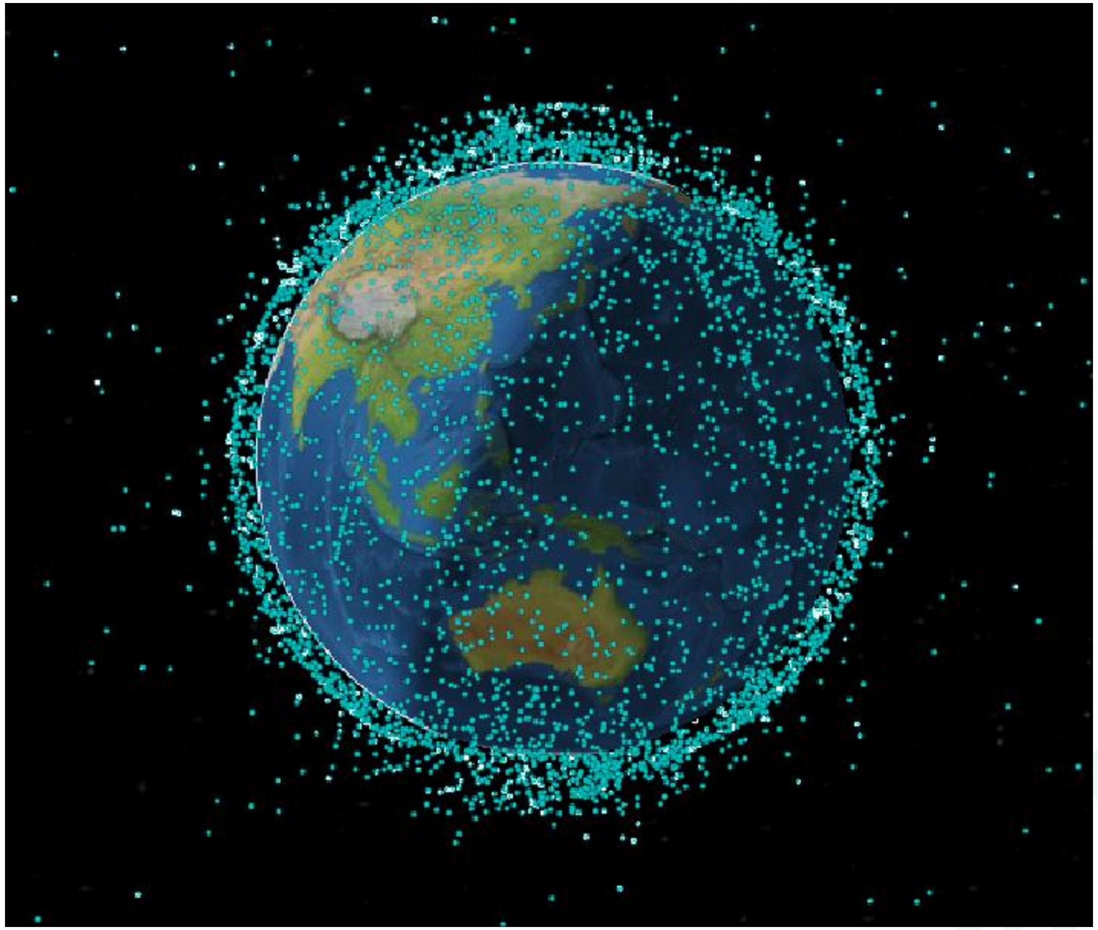
Artificial satellites

- All started with the launch of Sputink-1 on 4th October, 1957
- Ushered the space race and the space age
- Nations around the world launched fleet of artificial satellites to support critical services:
  - Telecommunication and broadcasting
  - Transport
  - Banking and finance
  - Weather forecast
  - Climate monitoring
  - Power grid
  - Remote sensing

Image curtesy: ISRO

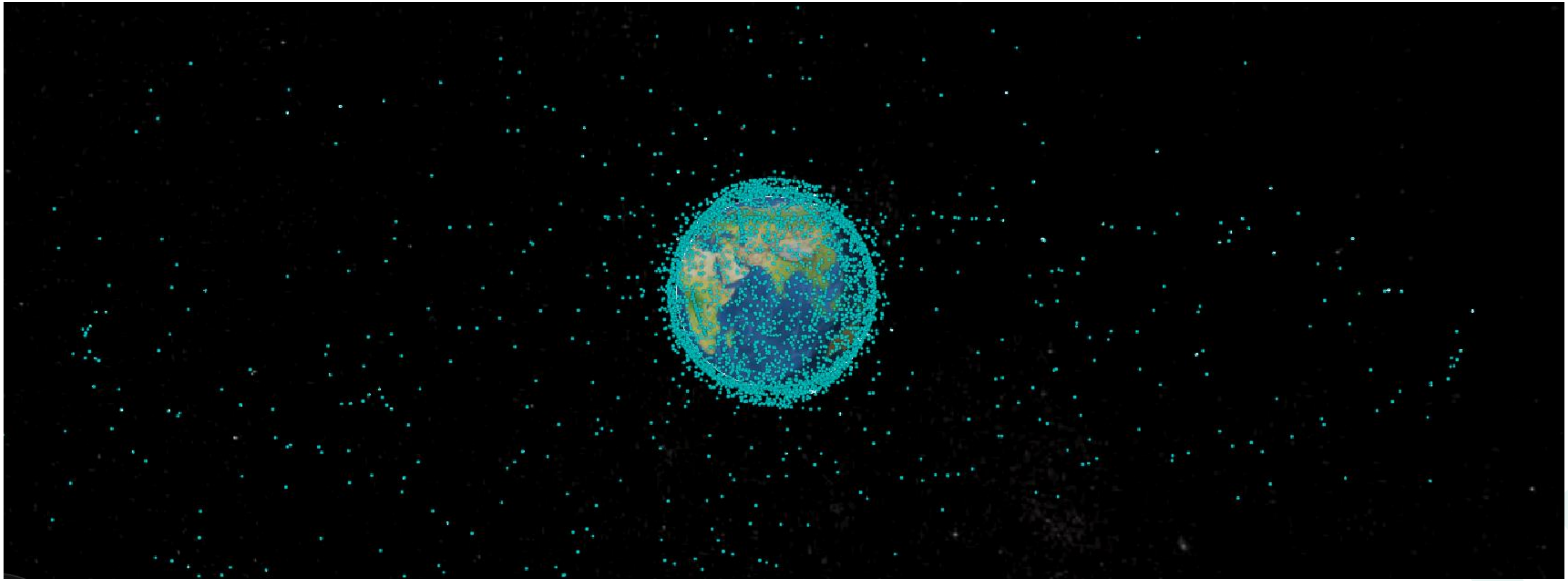
How many satellites are there in space?

- More than 20000 human-made space objects
- Number of active satellites 2,666
- 58 active satellites from India



https://orco.iiitd.edu.in/

Current scenario



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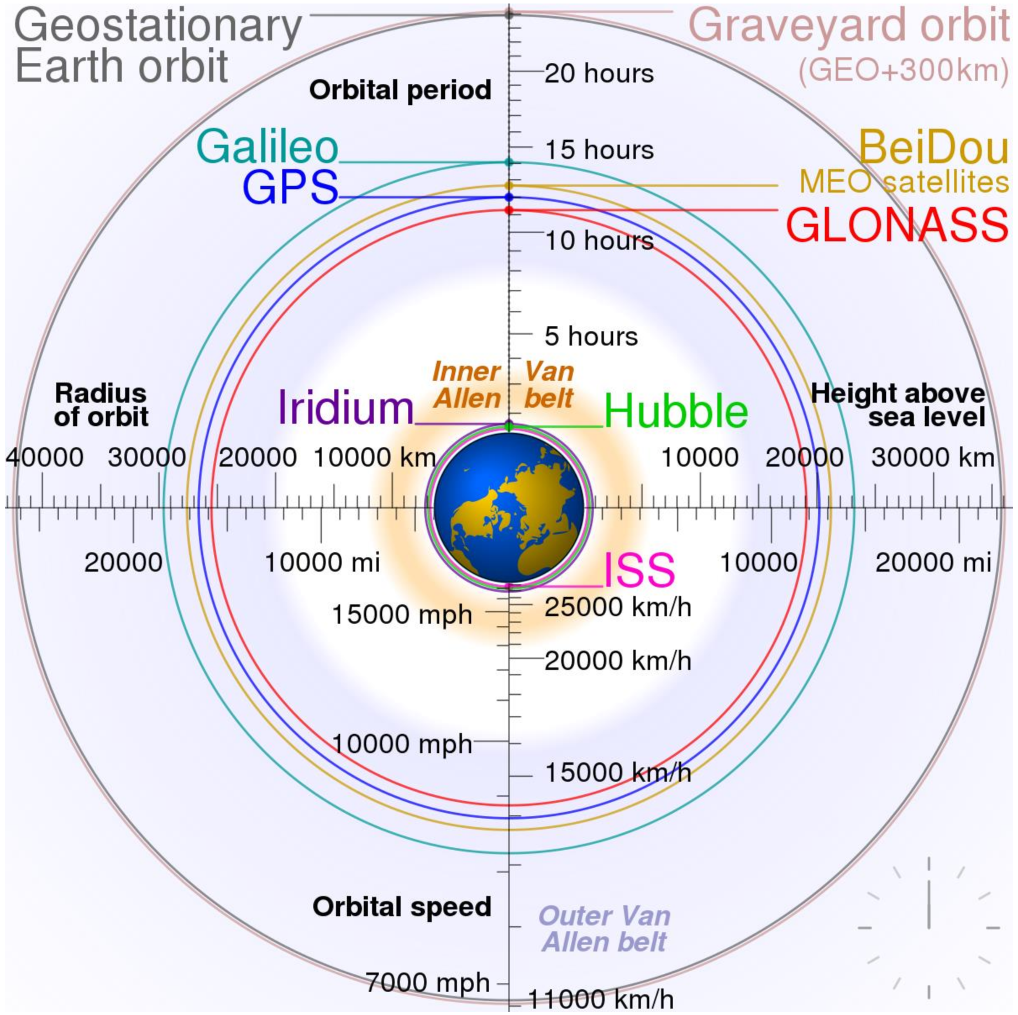
Various types of satellite orbits



https://upload.wikimedia.org/wikipedia/commons/b/b4/Comparison_satellite_navigation_orbits.svg

Altitude classification:

- Low Earth Orbit (LEO) - <2000km
- Medium Earth Orbit (MEO) - >2000 km and <35786 km
- Geo-synchronous Orbit (GSO) – at 35786 km (inclined)
- Geostationary Orbit (GEO) – at 35786 km (at equatorial plane)
- High Earth Orbit (HEO) - > 35786 km

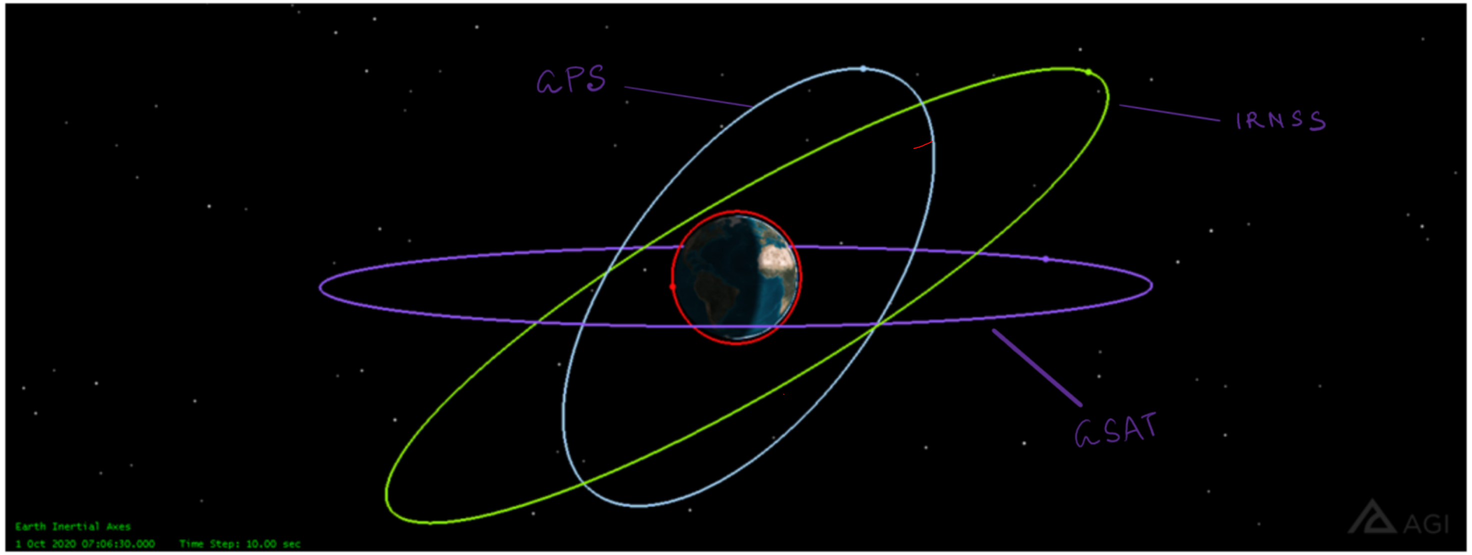
Inclination Classification

- Polar orbit – inclination is near 90 degree
- Sun-synchronous orbit – A polar orbit which preces around the earth's axis, satellite will be always at the same place at the same time of the day
- Ecliptic orbit – orbit plane on the ecliptic plane
- Equatorial Orbit – orbit plane on the equator plane

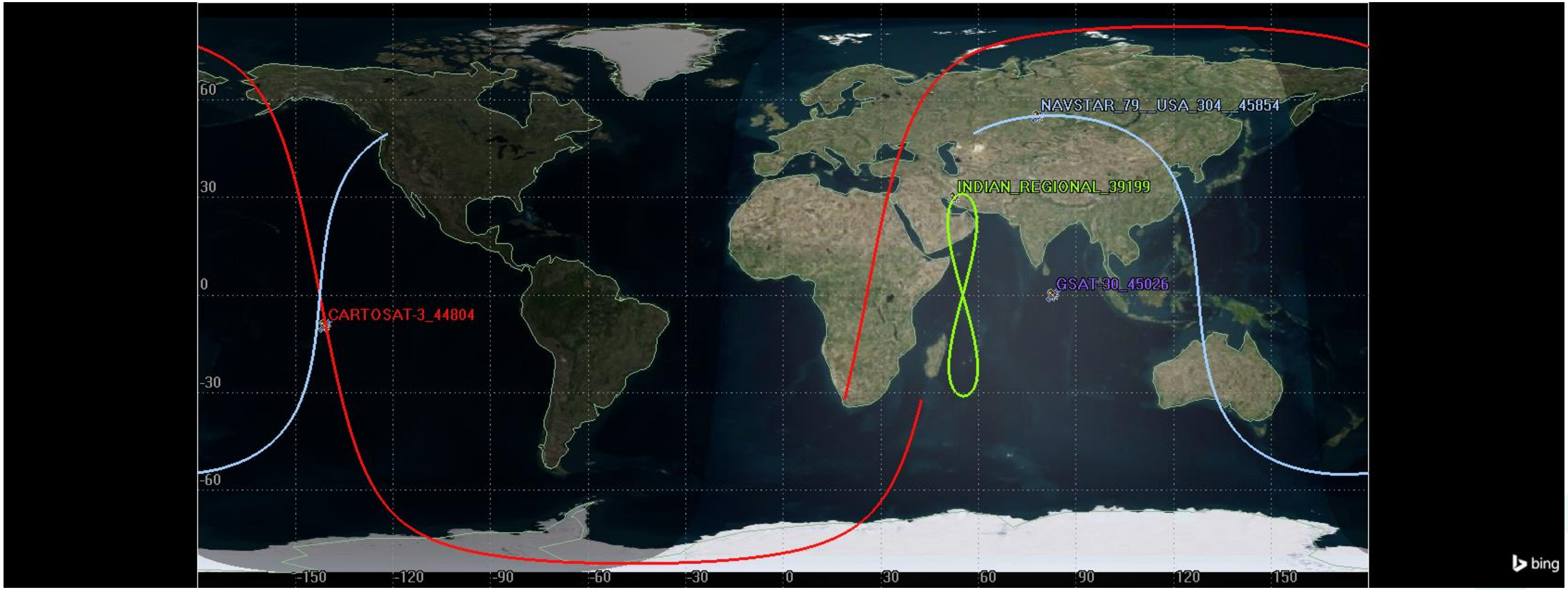
Directional classification

- Prograde orbit – satellite motion in the orbit is at the same direction as the earth's rotation
- Retrograde orbit – satellite motion is opposite to the direction of the earth's motion

Orbits in 3 dimensional space



Ground tracks



Orbital Mechanics

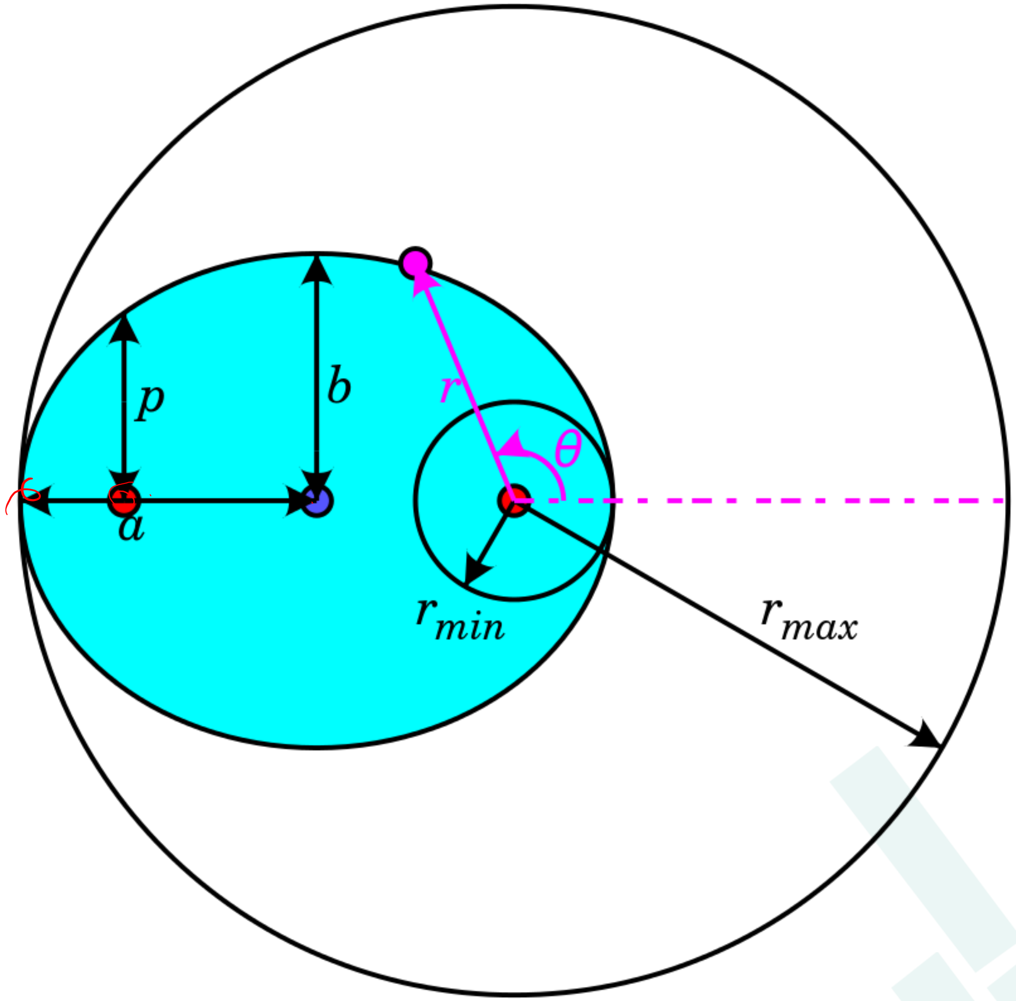
Orbital mechanics is the study of motion of space objects (natural or anthropogenic). From Satcom perspective, it will help you to answer the following questions:

- Where is a particular satellite?
- When will a satellite will be visible from a ground station?
- What should be the ground antenna pointing angles to receive signal from a satellite?
- What should be done to keep a satellite in a particular orbit (station keeping)?

Kepler's law

- **First Law:** The orbit of every planet is an ellipse with the Sun at one of the two foci.

a – semi-major axis
b – semi-minor axis
p – Latus rectum
θ – Angle from perigee



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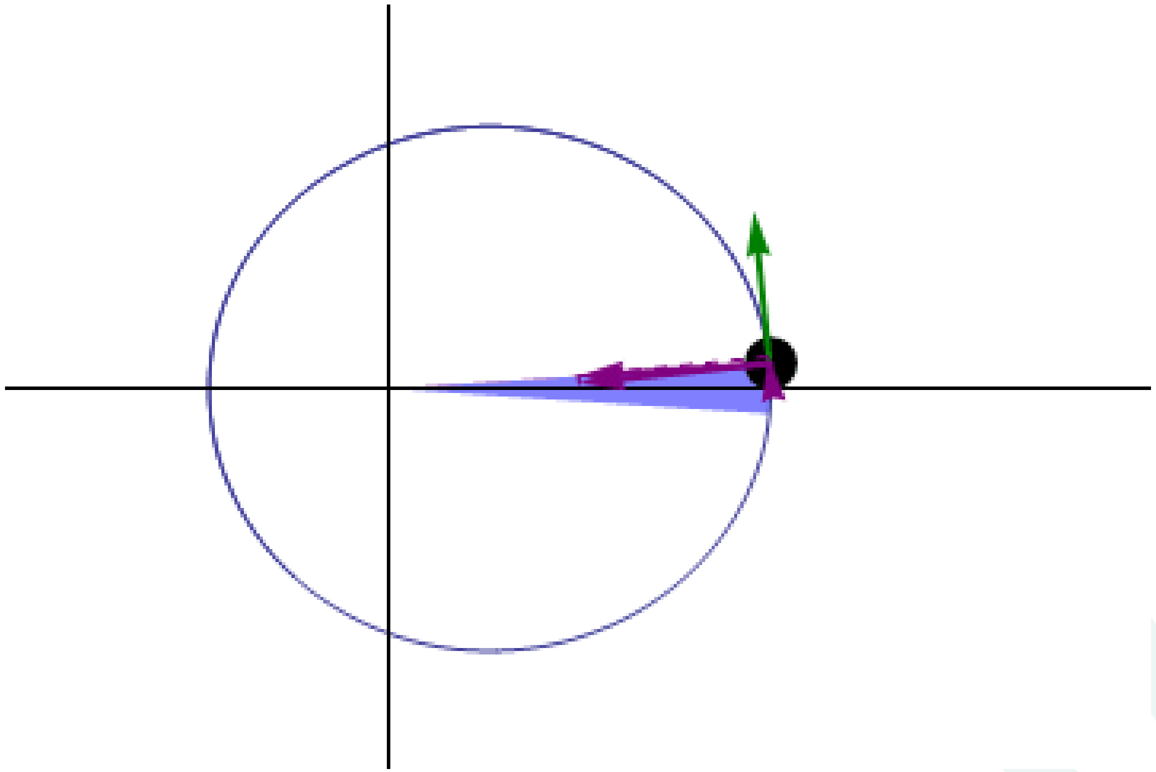
https://commons.wikimedia.org/w/index.php?curid=38314835

Kepler’s Law

Second Law: A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.



Kepler's law

- **Third Law:** The ratio of the square of an object's orbital period with the cube of the semi-major axis of its orbit is the same for all objects orbiting the same primary.

$$a^3 \propto T^2$$

Newton's Law of gravitation

The gravitational force between two masses M and m at a distance r between their centre of mass is:

$$F = -\frac{GMm}{r^2} \quad (1)$$

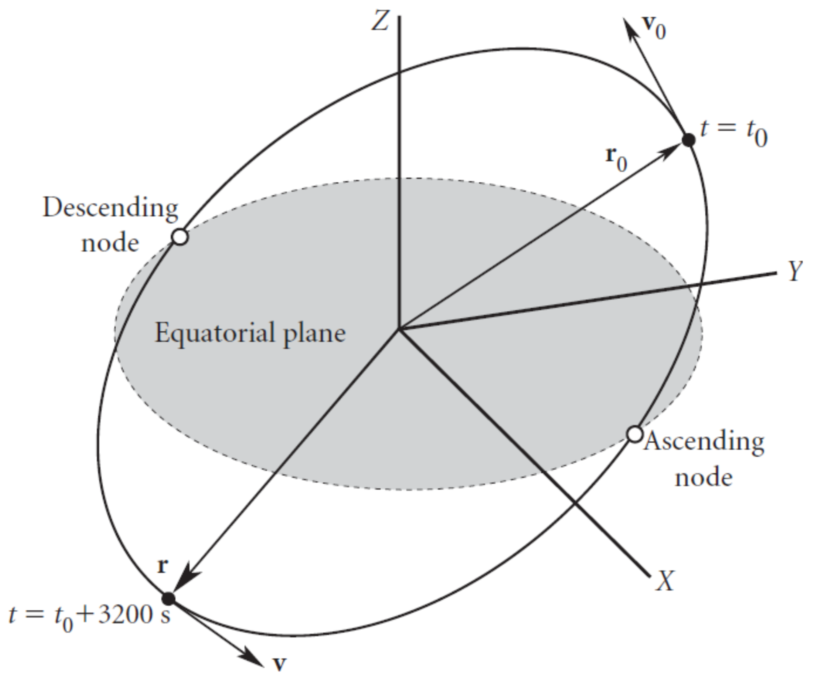
Or acceleration of m:

$$a = -\frac{GM}{r^2} \quad (2)$$

Equation of motion of a satellite in 3 dimension:

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} \quad (3)$$

Here, $\mu = GM$



Newton's Law of gravitation

- Starting from equation (2) Kepler's laws can be proved
- A satellite orbit in polar co-ordinate can be expressed as

$$r = \frac{h^2}{\mu}\frac{1}{1 + ecos\theta} \quad (4)$$

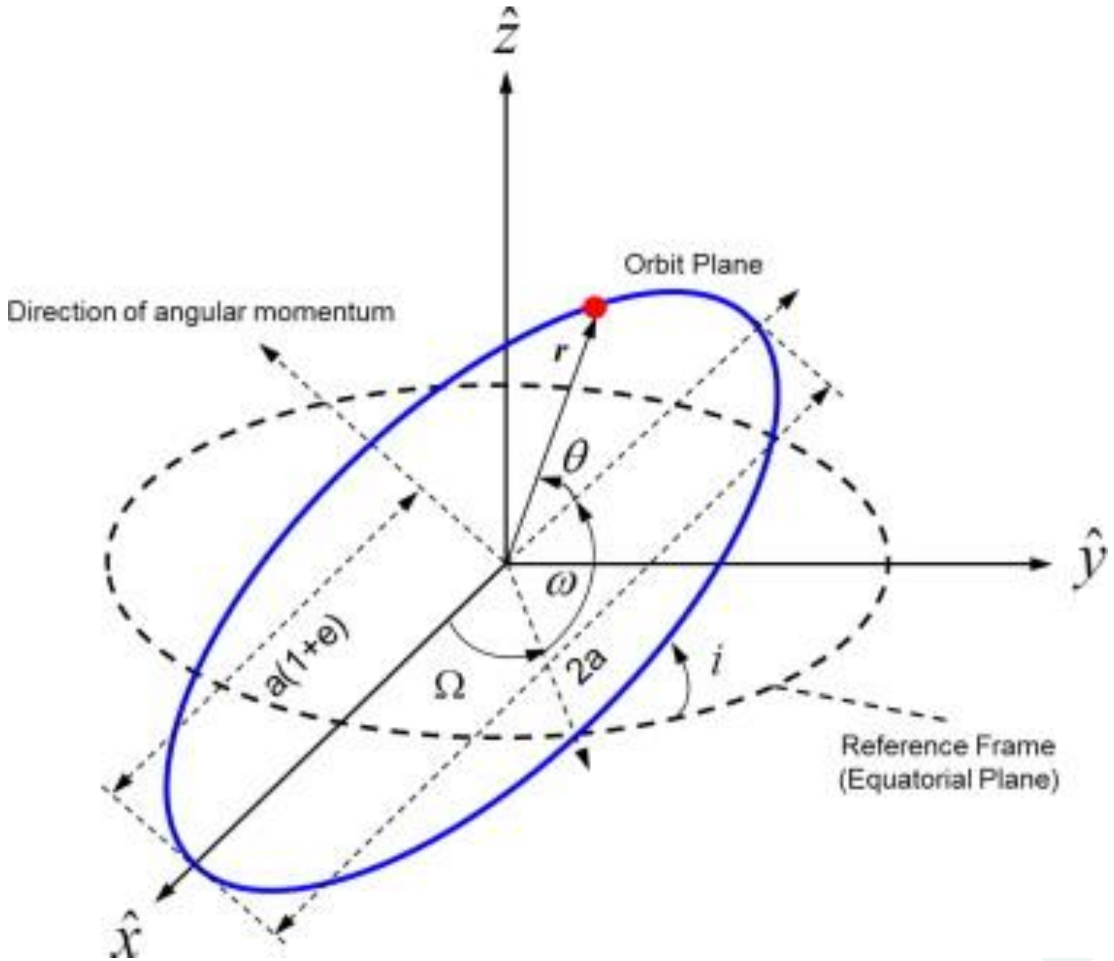
Which is equation of an ellipse

h = angular momentum magnitude

e = eccentricity

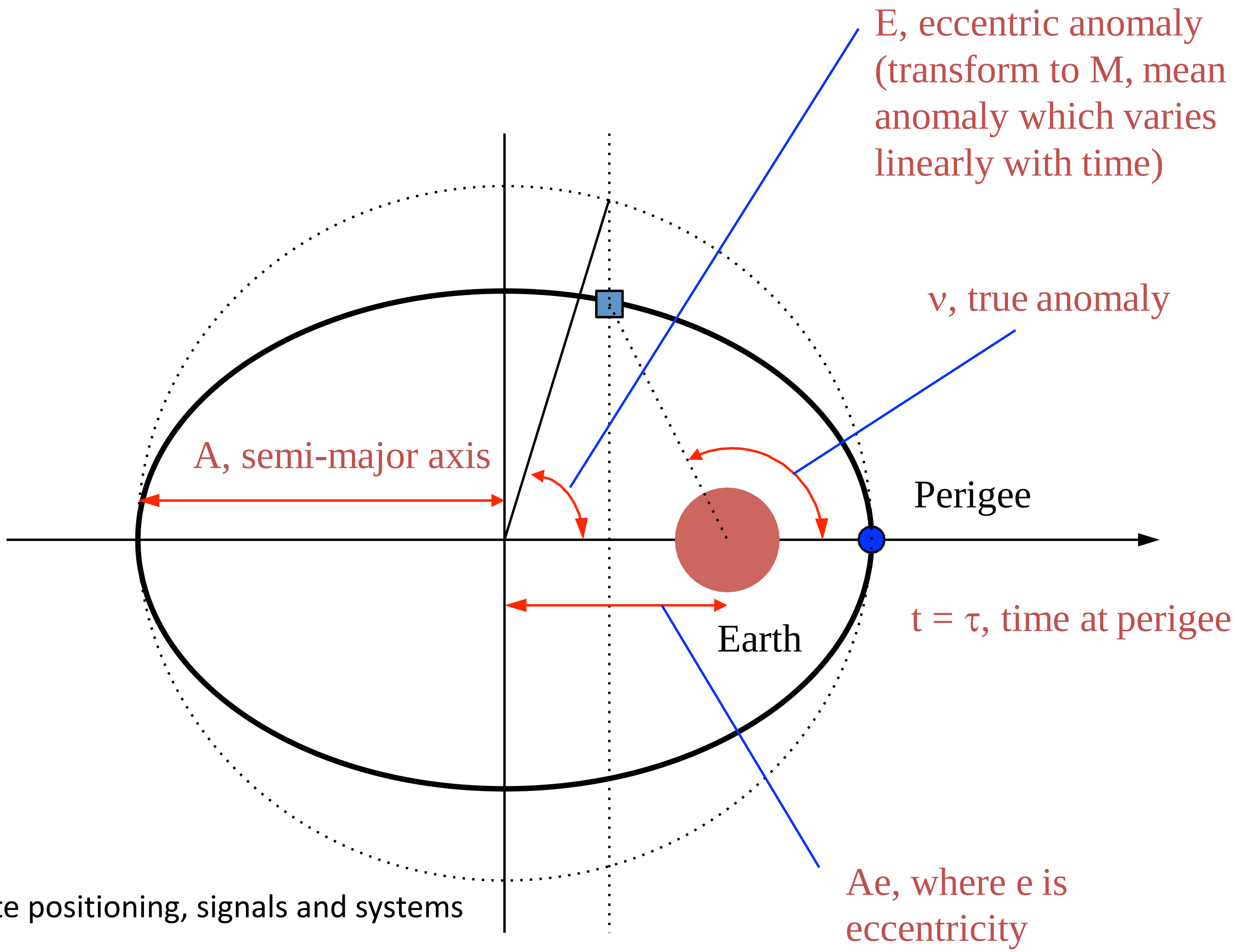
Orbital elements

- Semi major axis A
- Eccentricity e
- True anomaly θ
- Argument of perigee ω
- Inclination i
- Right ascension of ascending node Ω



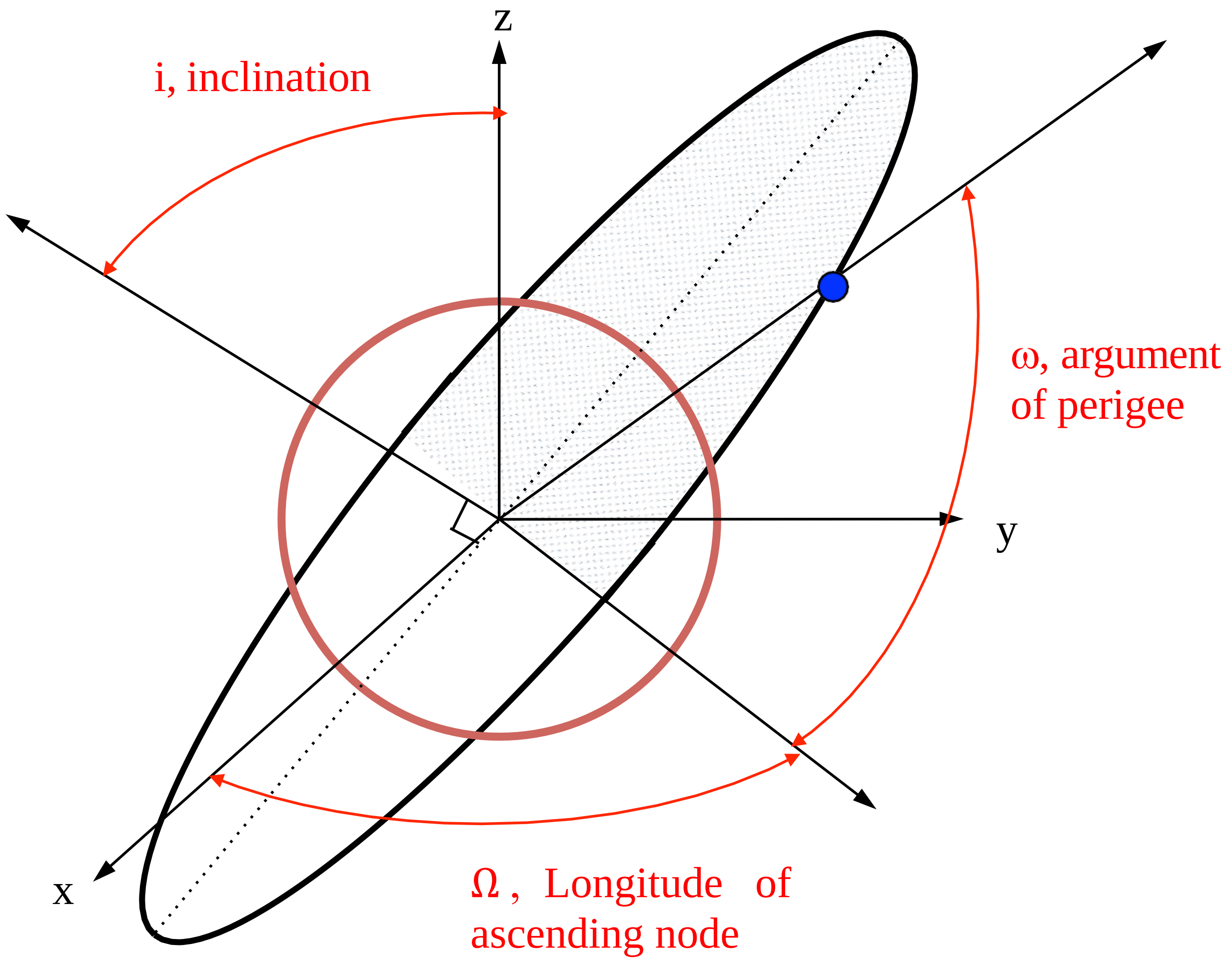
https://www.researchgate.net/publication/272198427_Radiation_effect_for_a_CubeSat_in_slow_transition_from_the_Earth_to_the_Moon/figures?lo=1

Keplerian Elements (shape)



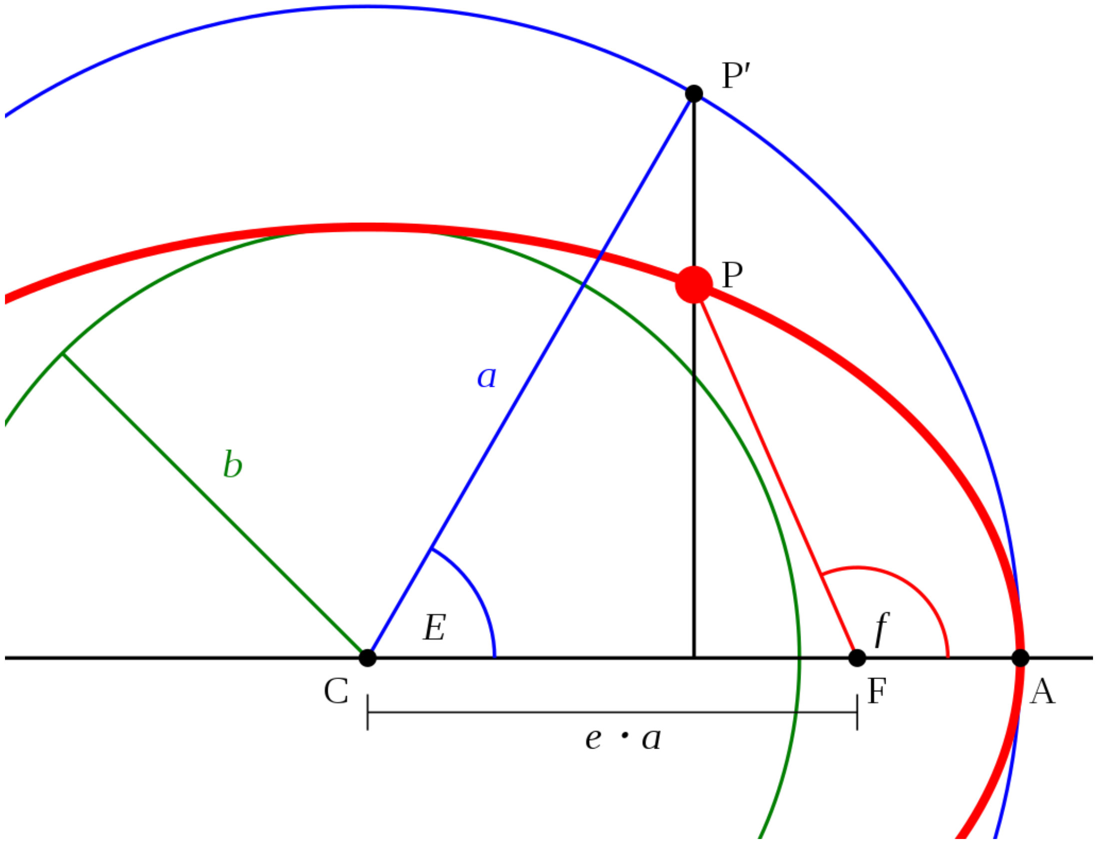
From lecture slides on Satellite positioning, signals and systems
By Prof. Andrew Dempster

Keplerian Elements (orientation)



From lecture slides on Satellite positioning, signals and systems
By Prof. Andrew Dempster

Anomalies



- True anomaly: f
- Eccentric Anomaly: E
- Mean anomaly (M):

$$n = \frac{360}{T}$$

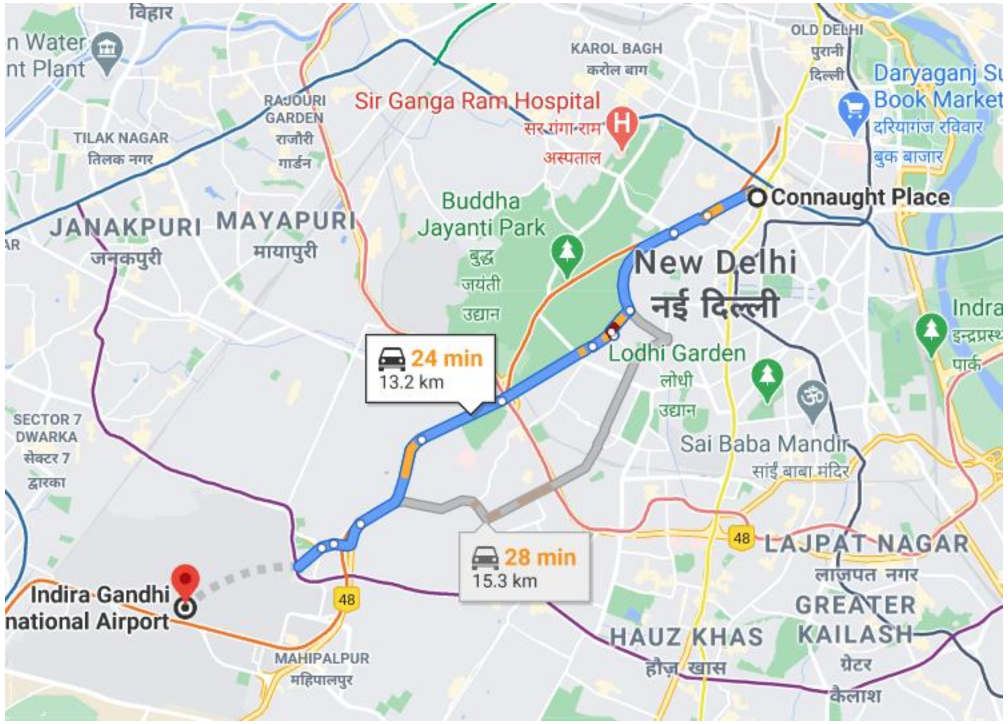
$$M = n(t - \tau)$$

$$M = E - esinE$$

$$tanE = \frac{\sqrt{1 - e^2}sin\theta}{1 + ecos\theta}$$

From lecture slides on Satellite positioning, signals and systems
By Prof. Andrew Dempster

Reference frames



Where is IGI airport?

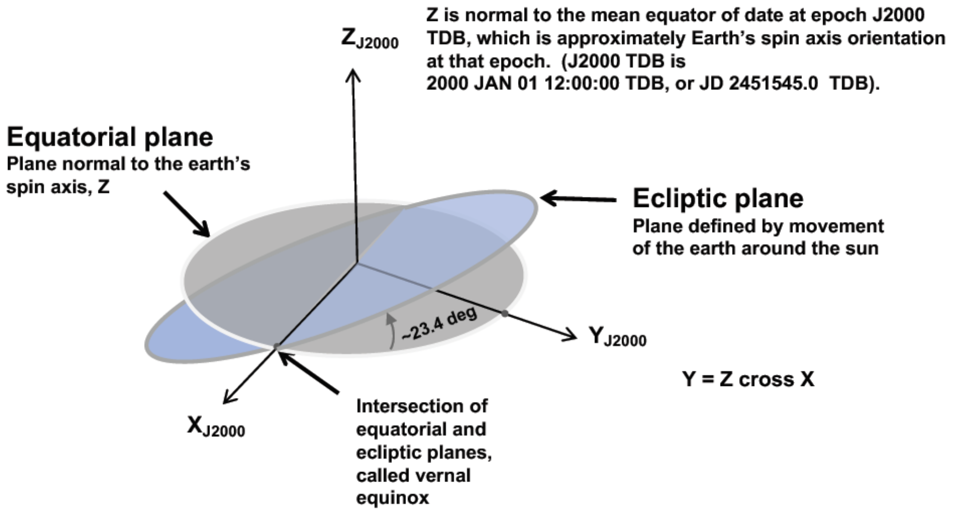
- From Connaught place (a known land mark) 13 km south-east
- Connaught place is the origin, south and east are directions – a reference frame!

Reference Frames

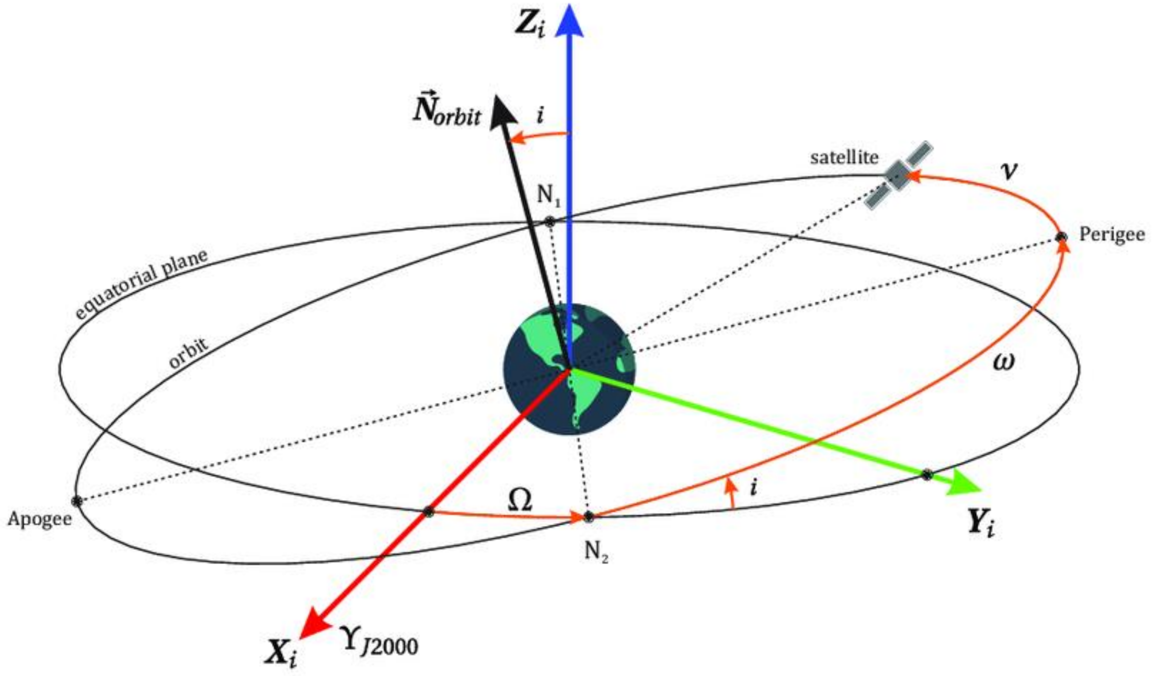
- Earth Centred Inertial (ECI)
- Earth Centred Earth Fixed (ECEF)
- Perifocal Frame
- Orbital frame
- Topocentric frame

Earth Centred Inertial

- Origin at the centre of the earth
- X axis towards the equinox on 1st January, 2000 at 12:00 Terrestrial Time (TT)
- Z axis is normal to the equatorial frame
- Y axis is determined by right hand rule

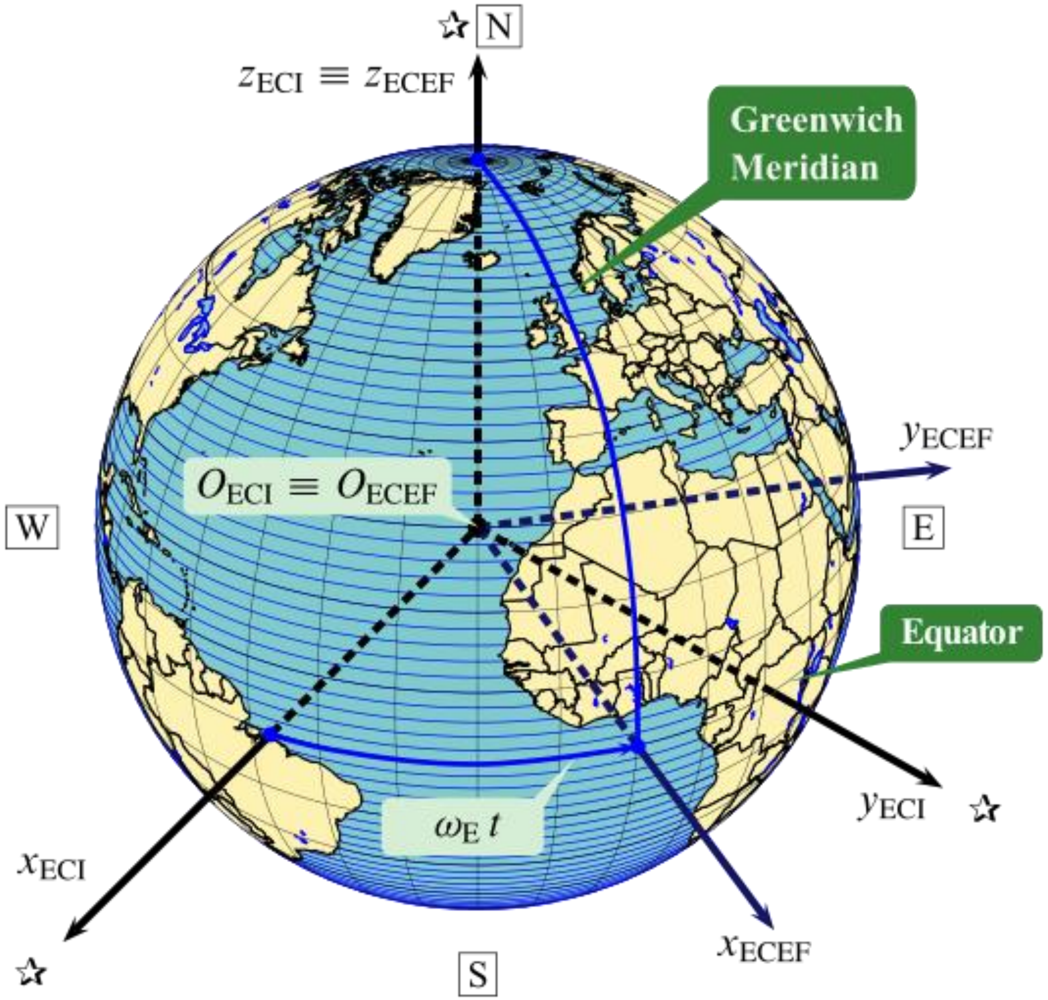


https://naif.jpl.nasa.gov/pub/naif/toolkit_docs/Tutorials/pdf/individual_docs/04_concepts.pdf



Macés, José Alfredo. (2017). Design and Analysis of the Attitude Control System for the S2TEP Mission.

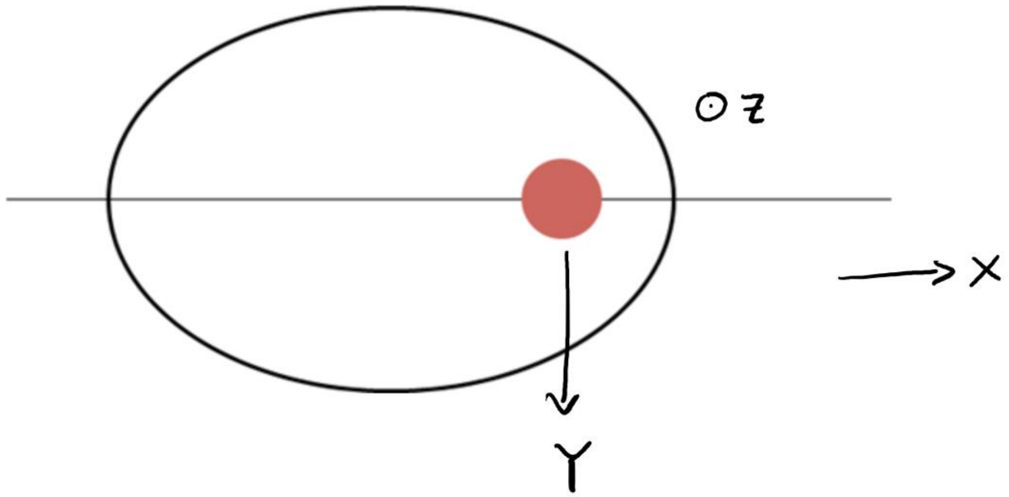
Earth Centred Earth Fixed



- Origin at the centre of the earth
- X axis towards the intersection of prime meridian and equator
- Z axis is normal to the equatorial frame
- Y axis is determined by right hand rule
- This is a non-inertial frame – rotates with respect to the ECI frame

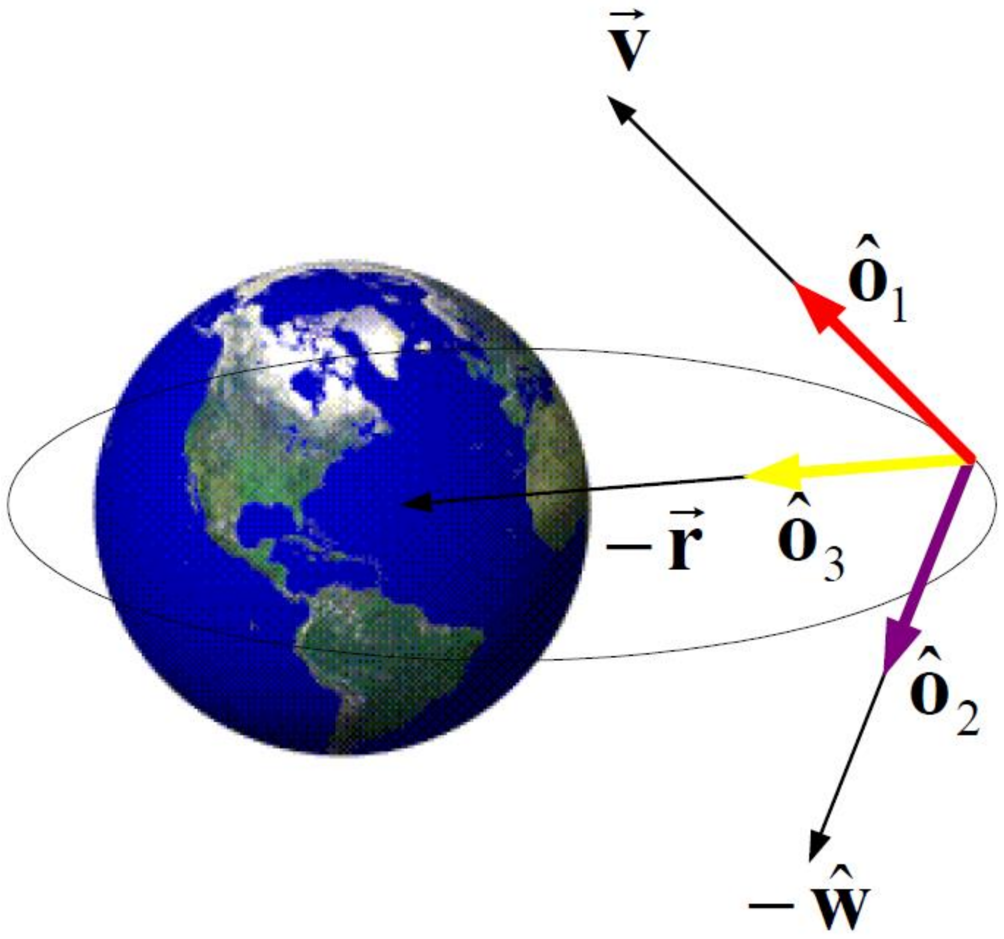
Perifocal frame

- Origin at the centre of the Earth
- X axis is towards the perigee
- Z axis is perpendicular to the orbital frame (towards angular momentum vector)
- Y axis is defined by the right hand rule



Orbital frame

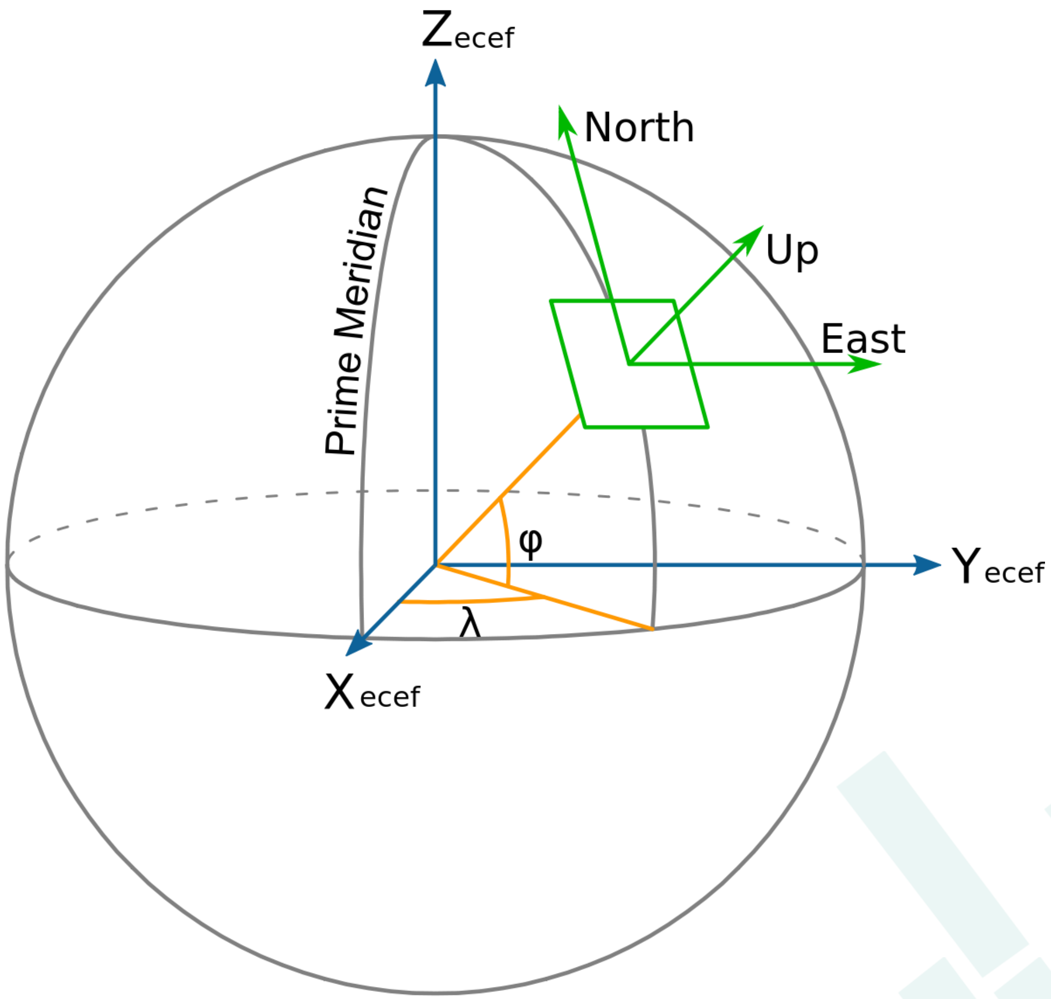
- O_1 towards the velocity vector
- O_3 towards the centre of the earth (nadir direction)
- O_2 towards negative to orbit normal (direction of the angular momentum vector)



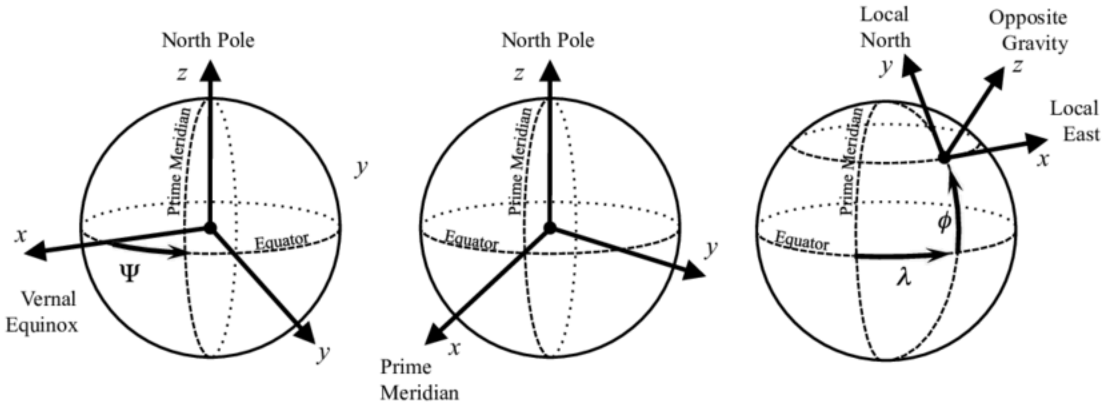
Reference Frames for Spacecraft Dynamics and Control, Chris Hall

Topocentric frame

- Local frame
- X/ Y directions are towards local North/ East
- Z direction is towards nadir or zenith (up/ down)
- Various combinations can be considered, most popular – ENU (XYZ), NED (XYZ)



ECI, ECEF and Topocentric



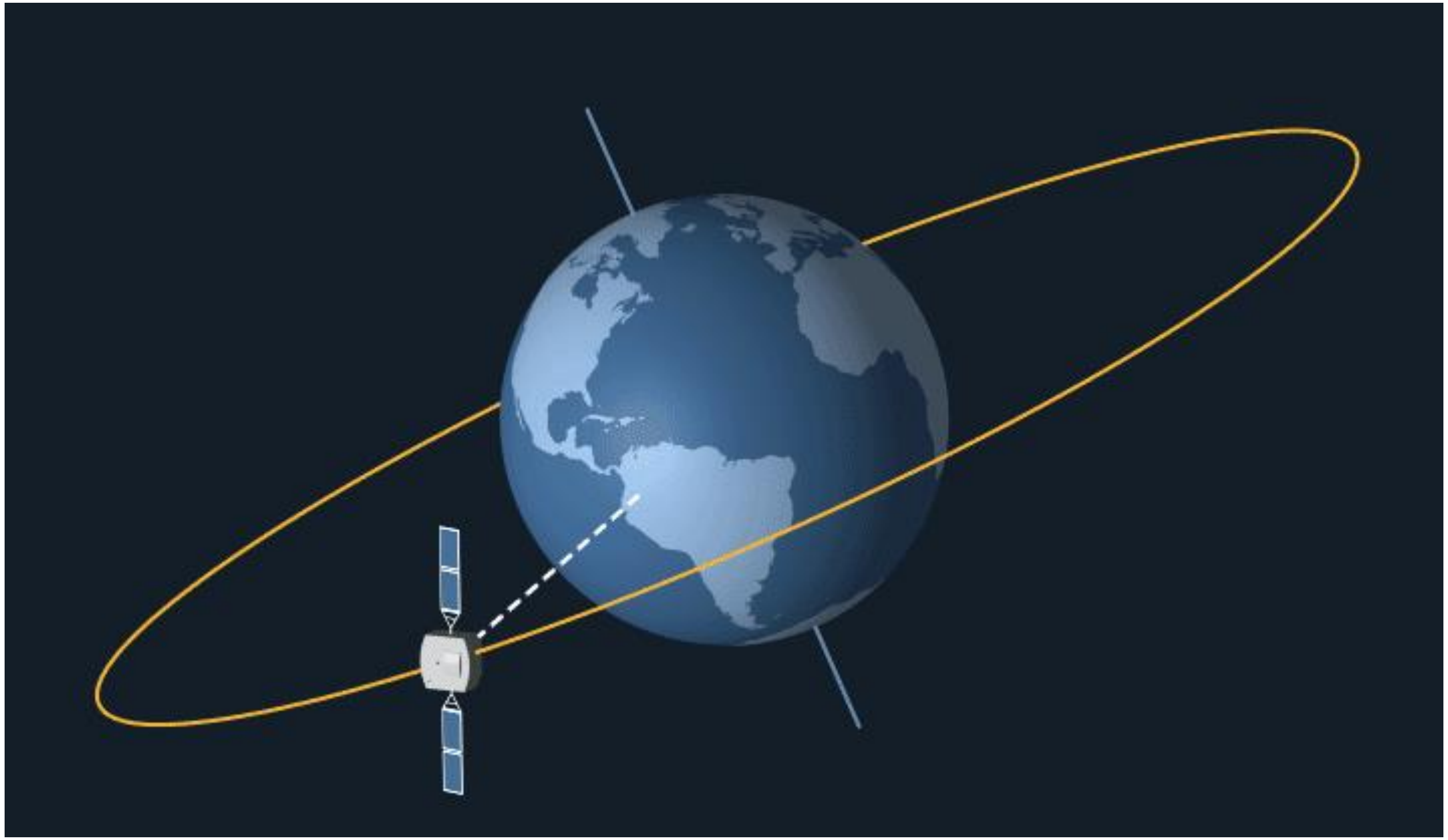
EARTH CENTRED INERTIAL: *I* EARTH CENTRED FIXED: *F* TOPOCENTRIC: *T*

Furgale, Paul & Enright, John & Barfoot, Timothy. (2011). Sun Sensor Navigation for Planetary Rovers: Theory and Field Testing. Aerospace and Electronic Systems, IEEE Transactions on. 47. 1631 - 1647. 10.1109/TAES.2011.5937255.

Practical orbits

- Apart from gravitational force, other forces act on a satellite:
  - Atmospheric drag
  - Solar radiation pressure
  - Gravitational force from other celestial objects
- Orbit derived from Newton's Law of Gravitation assumes uniform density of the earth and considers the earth is perfect sphere, which is not the reality!
- Additional correction terms must be added in the equation to obtain actual satellite position in the space
- All these additional forces and force correction due to gravitational field variations due to non-uniform structure of the earth are called perturbation forces (perturbation from ideal)

Geostationary satellites



https://solarsystem.nasa.gov/basics/chapter5-1/

Calculation of Geostationary altitude

Gravitational force:

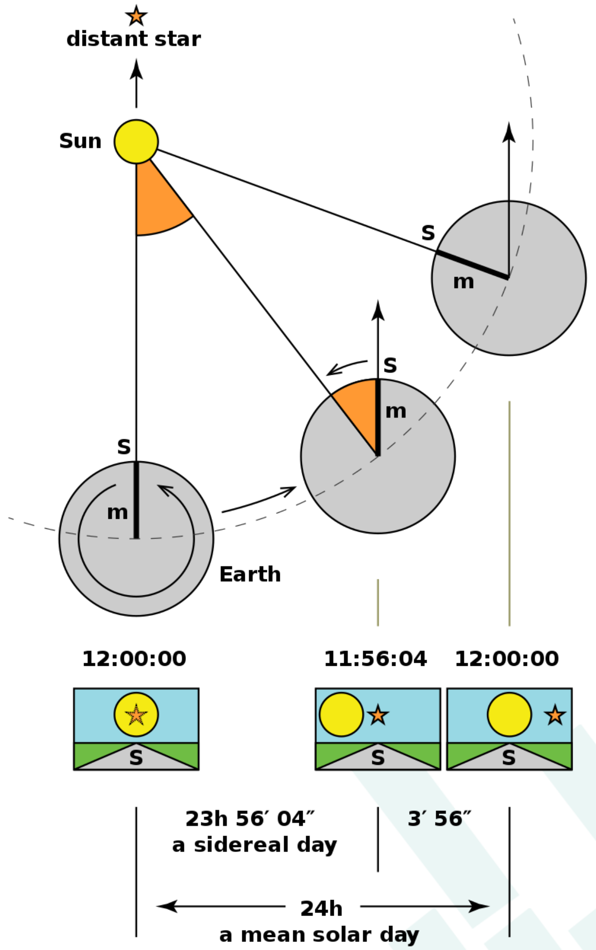
$$F_g = \frac{GMm}{r^2}$$

Force acting on the satellite according to Newton's second law:

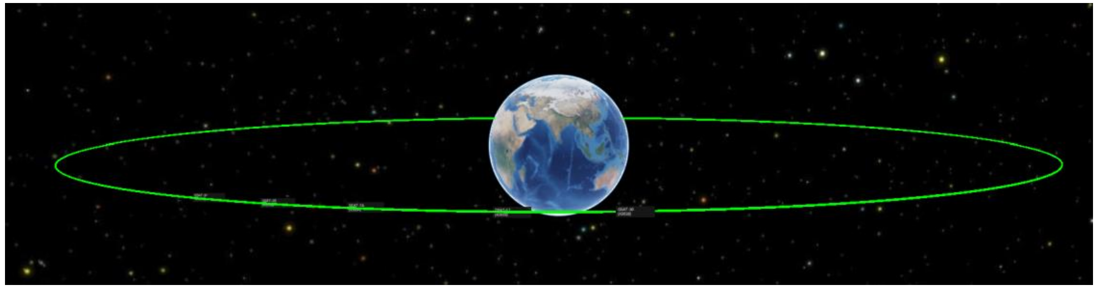
$$F_C = \frac{mv^2}{r} \qquad v^2 = \frac{GM}{r}$$

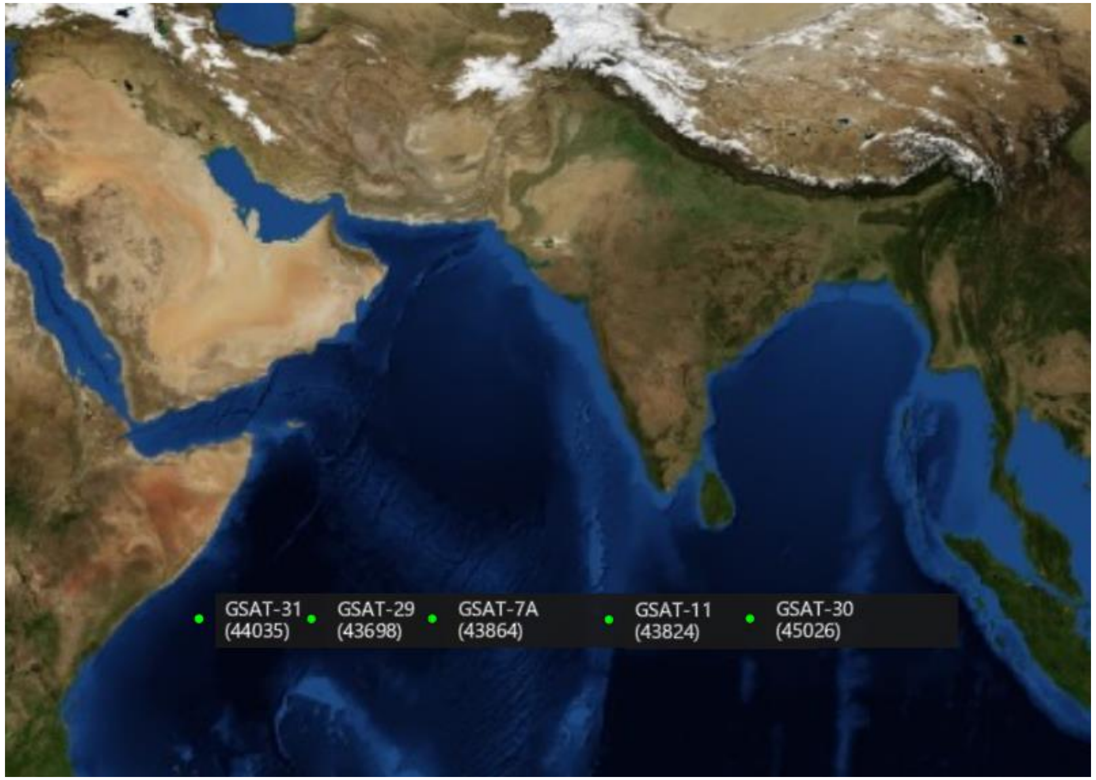
$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \qquad r = \sqrt[3]{\frac{\mu T^2}{4\pi^2}}$$

Orbital period – 1 sidereal day = 23 hr 56 m 4 s
Orbit radius r is 42,164 kilometres



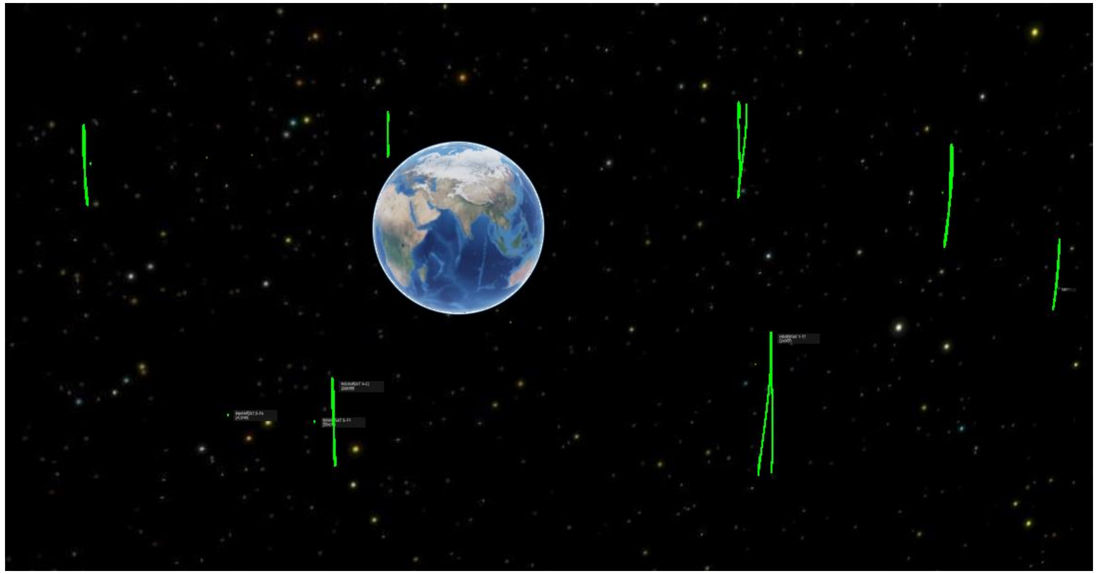
GSAT





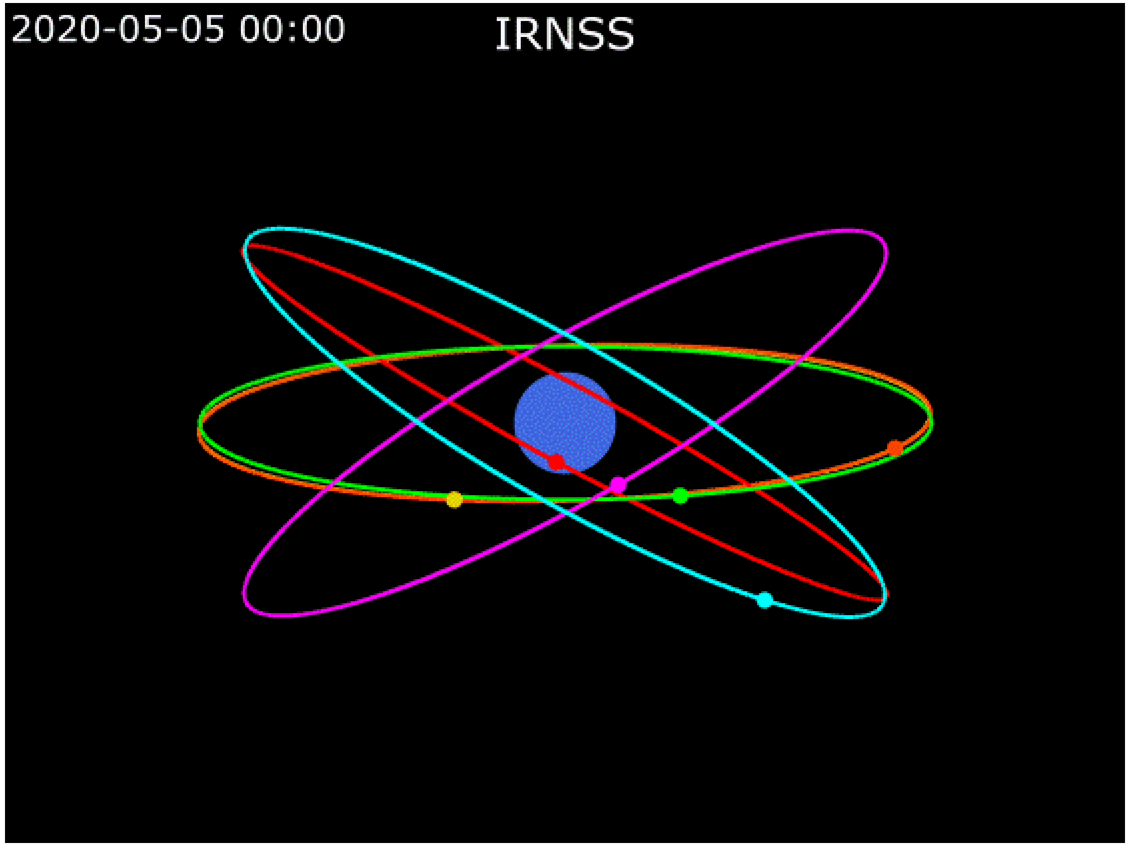
- Successor of INSAT system
- Geostationary satellites
- Used for:
  - Mobile communication
  - Military and naval applications
  - SBAS services
  - DTH, Tele-education, telemedicine
  - Maritime services

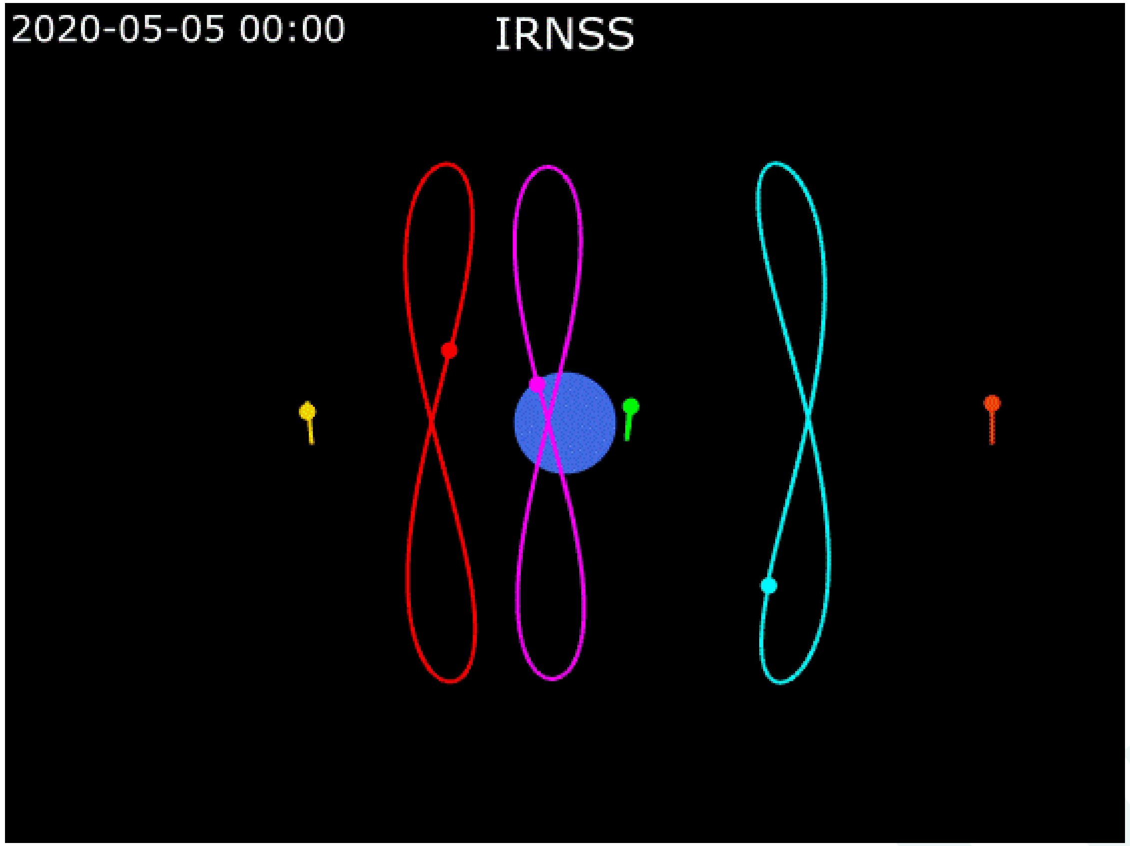
INMARSAT



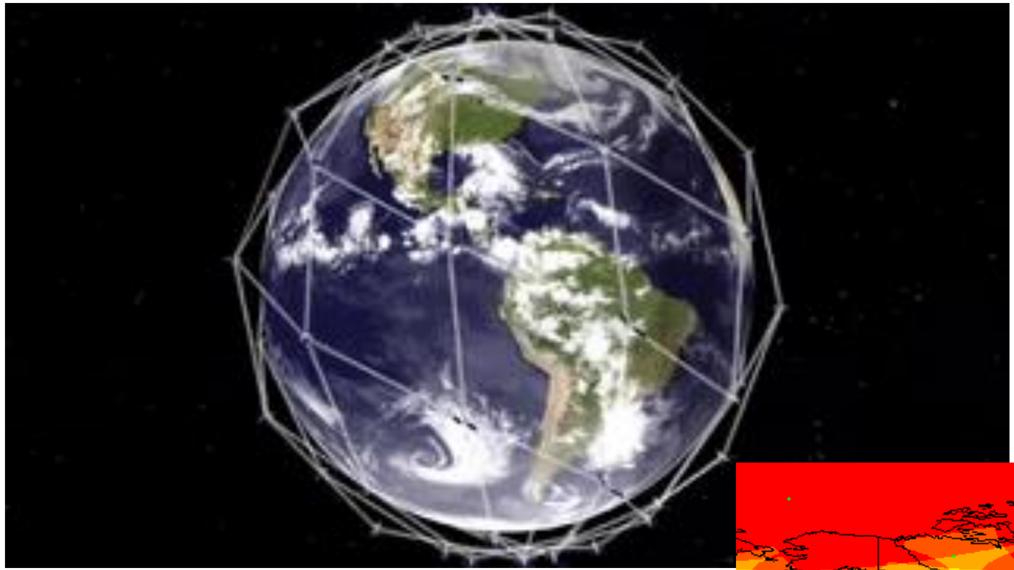
- Provides communication services globally
- User segment can be mobile or stationary terminals
- Heavily used where reliable terrestrial network services are not available
- Use case: Maritime distress services

IRNSS (NavIC) Constellation



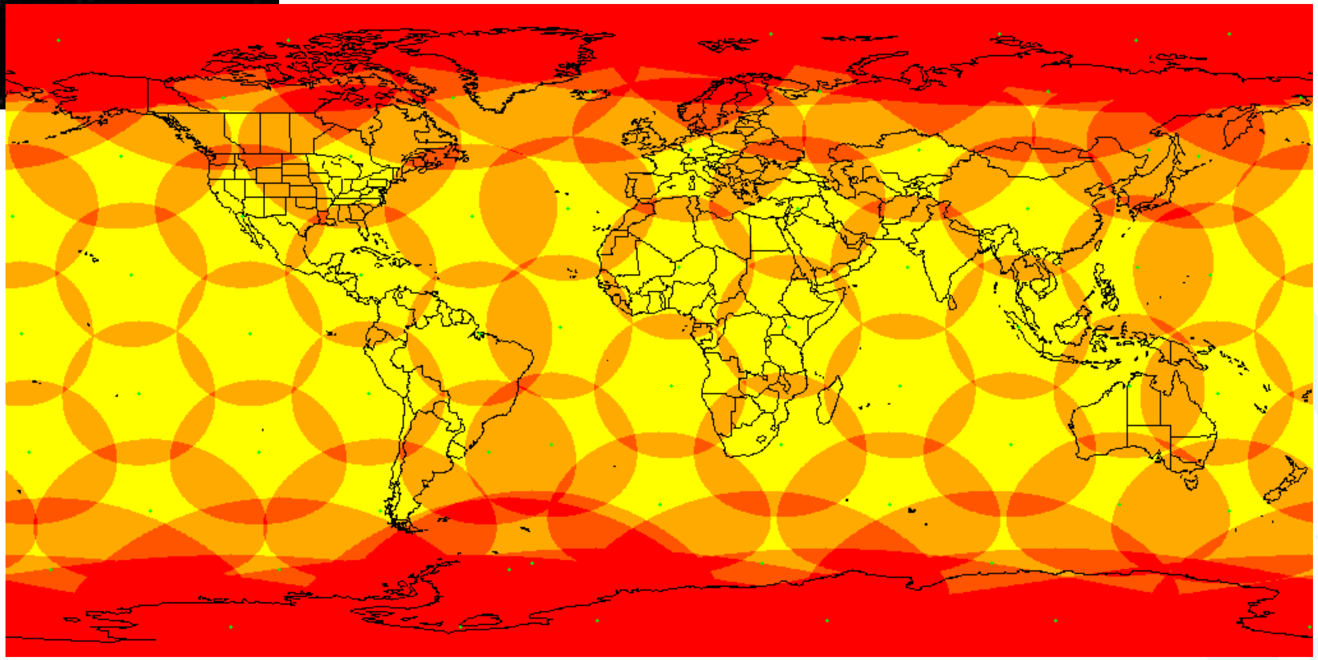


Iridium Constellation

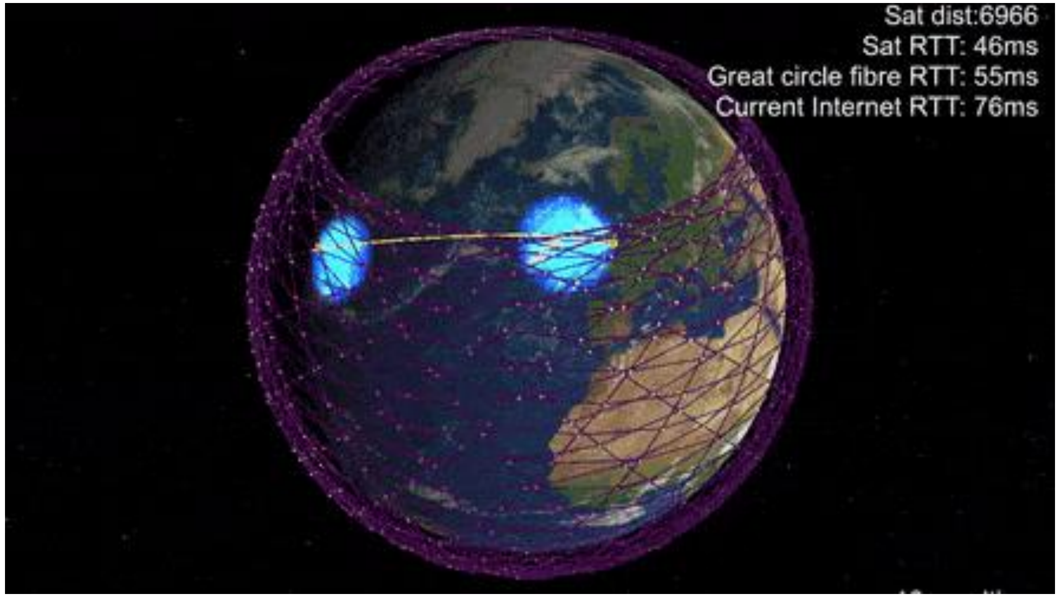


- LEO based communication constellation
- Provides subscription based services
- Aims for global coverage
- Now certified for Maritime distress services

Source: Internet



Mega Constellation - Starlink



Source: Internet

- Aims for global internet coverage
- Satellite to consumer model
- LEO constellation
- Requirement of low latency requires a massive number of satellites – 1440 planned

Conclusion

- Various types of orbits are used for space-based communications
- Choice of orbit is a trade off subject to regulatory constraints
- Orbital mechanics is used to calculate and predict satellite position in space
- With increased demand of data, space-based communication has become extremely important, also increased the complexity